OCR
RECOGNISING ACHIEVEMENT

Friday 27 January 2012 – Morning

A2 GCE MATHEMATICS

4727 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The variables x and y are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x^2 + y^2}{xy}. \qquad \text{(A)}$$

(i) Use the substitution $y = ux$, where u is a function of x, to obtain the differential equation

$$x\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2}{u}.$$ **[3]**

(ii) Hence find the general solution of the differential equation (A), giving your answer in the form $y^2 = \mathrm{f}(x)$. **[4]**

2 **(i)** Show that $(z^n - \mathrm{e}^{\mathrm{i}\theta})(z^n - \mathrm{e}^{-\mathrm{i}\theta}) \equiv z^{2n} - (2\cos\theta)z^n + 1$. **[1]**

(ii) Express $z^4 - z^2 + 1$ as the product of four factors of the form $(z - \mathrm{e}^{\mathrm{i}\alpha})$, where $0 \leqslant \alpha < 2\pi$. **[6]**

3 A multiplicative group contains the distinct elements e, x and y, where e is the identity.

(i) Prove that $x^{-1}y^{-1} = (yx)^{-1}$. **[2]**

(ii) Given that $x^n y^n = (xy)^n$ for some integer $n \geqslant 2$, prove that $x^{n-1}y^{n-1} = (yx)^{n-1}$. **[3]**

(iii) If $x^{n-1}y^{n-1} = (yx)^{n-1}$, does it follow that $x^n y^n = (xy)^n$? Give a reason for your answer. **[2]**

4 The line l has equations $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{2}$ and the point A is $(7, 3, 7)$. M is the point where the perpendicular from A meets l.

(i) Find, in either order, the coordinates of M and the perpendicular distance from A to l. **[7]**

(ii) Find the coordinates of the point B on AM such that $\overrightarrow{AB} = 3\overrightarrow{BM}$. **[3]**

5 The variables x and y satisfy the differential equation

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 5\mathrm{e}^{-2x}.$$

(i) Find the complementary function of the differential equation. **[2]**

(ii) Given that there is a particular integral of the form $y = px\mathrm{e}^{-2x}$, find the constant p. **[4]**

(iii) Find the solution of the equation for which $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ when $x = 0$. **[5]**

6 The plane Π has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -5 \end{pmatrix}$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$.

(i) Express the equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = p$. **[4]**

(ii) Find the point of intersection of l and Π. **[2]**

(iii) The equation of Π may be expressed in the form $\mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \mu\mathbf{c}$, where $\mathbf{c}$ is perpendicular to $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$. Find $\mathbf{c}$. **[3]**

7 The set M consists of the six matrices $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$, where $n \in \{0, 1, 2, 3, 4, 5\}$. It is given that M forms a group $(M, \times)$ under matrix multiplication, with numerical addition and multiplication both being carried out modulo 6.

(i) Determine whether $(M, \times)$ is a commutative group, justifying your answer. **[2]**

(ii) Write down the identity element of the group and find the inverse of $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. **[3]**

(iii) State the order of $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ and give a reason why $(M, \times)$ has no subgroup of order 4. **[2]**

(iv) The multiplicative group G has order 6. All the elements of G, apart from the identity, have order 2 or 3. Determine whether G is isomorphic to $(M, \times)$, justifying your answer. **[2]**

8 **(i)** Use de Moivre's theorem to prove that

$$\tan 5\theta \equiv \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$ **[4]**

(ii) Solve the equation $\tan 5\theta = 1$, for $0 \leqslant \theta < \pi$. **[3]**

(iii) Show that the roots of the equation

$$t^4 - 4t^3 - 14t^2 - 4t + 1 = 0$$

may be expressed in the form $\tan\alpha$, stating the exact values of α, where $0 \leqslant \alpha < \pi$. **[5]**

Question			Answer	Marks	Guidance
1	(i)		$(y = xu \Rightarrow)\ \frac{dy}{dx} = x\frac{du}{dx} + u$	B1	For a correct statement
			$x\frac{du}{dx} + u = \frac{2+u^2}{u}$	M1	For using the substitution to eliminate y (If B0, then y must be eliminated from LHS, but $\frac{d(uv)}{dx}$ sufficient)
			$\Rightarrow\ x\frac{du}{dx} = \frac{2}{u}$	A1	For correct equation **AG**
				[3]	
1	(ii)		$\int u\ du = \int \frac{2}{x}\ dx$	M1	For separating variables and writing/attempting integrals
			$\Rightarrow\ \frac{1}{2}u^2 = 2\ln((k)x)\ \ OR\ \ \frac{1}{2}u^2 = 2\ln x\ (+c)$	A1	For correct integration both sides (k or c not required here)
			$\Rightarrow\ \frac{1}{2}\left(\frac{y}{x}\right)^2 = 2\ln(kx)\ \ OR\ \ \frac{1}{2}\left(\frac{y}{x}\right)^2 = 2\ln x + c$	M1	For substituting for u into integrated terms with constant (on either side)
			$\Rightarrow\ y^2 = 4x^2\ln(kx)\ \ OR\ \ y^2 = 4x^2\ln x + Cx^2$	A1	For correct solution **AEF** $y^2 = f(x)$ Do not penalise "c" being used for different constants e.g. $2\ln x + c = 2\ln(cx)$
				[4]	
2	(i)		$\left(z^n - e^{i\theta}\right)\left(z^n - e^{-i\theta}\right) \equiv z^{2n} - 2z^n\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right) + 1$ $\equiv z^{2n} - (2\cos\theta)z^n + 1$	B1	For multiplying out to **AG** with evidence of $\cos\theta = \frac{1}{2}\left(e^{i\theta}+e^{-i\theta}\right)$ (Can be implied by $2\cos\theta = \left(e^{i\theta}+e^{-i\theta}\right)$)
				[1]	

Question			Answer	Marks	Guidance
2	(ii)		METHOD 1 $2\cos\theta = 1 \Rightarrow \theta = \frac{1}{3}\pi$ $\Rightarrow z^4 - z^2 + 1 \equiv \left(z^2 - e^{\frac{1}{3}\pi i}\right)\left(z^2 - e^{-\frac{1}{3}\pi i}\right)$ $\equiv \left(z + e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{1}{6}\pi i}\right)\left(z + e^{-\frac{1}{6}\pi i}\right)\left(z - e^{-\frac{1}{6}\pi i}\right)$ $\equiv \left(z - e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{5}{6}\pi i}\right)\left(z - e^{\frac{7}{6}\pi i}\right)\left(z - e^{\frac{11}{6}\pi i}\right)$	M1 A1 M1 A1 M1 A1 **[6]**	For using **(i)** to find θ For correct quadratic factors (Or $\frac{5\pi}{3}i$ in place of $-\frac{\pi}{3}i$) For factorising $\left(z^2 - a^2\right)$ For correct linear factors For adjusting arguments (must attempt correct range **and** "(z – root)") For correct factors **CAO** Correct answer www gets 6
			METHOD 2 $z^4 - z^2 + 1 = 0 \Rightarrow z^2 = \frac{1}{2} \pm \frac{1}{2}\sqrt{3}\,i = e^{\frac{1}{3}\pi i},\ e^{-\frac{1}{3}\pi i}$ $\Rightarrow z = \pm e^{\frac{1}{6}\pi i},\ \pm e^{-\frac{1}{6}\pi i}$ $= e^{\frac{1}{6}\pi i},\ e^{\frac{7}{6}\pi i},\ e^{\frac{5}{6}\pi i},\ e^{\frac{11}{6}\pi i}$ $\Rightarrow \left(z - e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{5}{6}\pi i}\right)\left(z - e^{\frac{7}{6}\pi i}\right)\left(z - e^{\frac{11}{6}\pi i}\right)$	M1 A1 M1 A1 M1 A1	For solving quadratic For correct roots in exp form For attempt to find 4 roots For correct roots $\pm e^{i\alpha}$ For adjusting arguments For correct factors **CAO**
3	(i)		METHOD 1 $(yx)(yx)^{-1} = e \Rightarrow x(yx)^{-1} = y^{-1}$ $\Rightarrow (yx)^{-1} = x^{-1}y^{-1}$ METHOD 2 Compare $(yx)(yx)^{-1} = e$ with $yxx^{-1}y^{-1} = e$ $\Rightarrow (yx)^{-1} = x^{-1}y^{-1}$	M1 A1 **[2]** M1 A1	For starting point and appropriate multiplication For correct result **AG** For appropriate comparison For correct result **AG** For A1, proof cannot be written in the form 'LHS = RHS $\rightarrow \ldots \rightarrow$ e = e'

Question			Answer	Marks	Guidance
3	(ii)		$x^n y^n = (xy)^n = x(yx)^{n-1}y$ $\Rightarrow x^{-1}x^n y^n y^{-1} = x^{-1}x(yx)^{n-1}yy^{-1}$ $\Rightarrow x^{n-1}y^{n-1} = (yx)^{n-1}$	M1 M1 A1 **[3]**	For using associativity or an inverse with respect to LHS, RHS or initial equality **www beforehand** For using $(xy)^n = x(yx)^{n-1}y$ **oe** For correct result **AG** **SR** for numerical *n* used, allow M1 M1 only
3	(iii)		METHOD 1 All steps in **(ii)** are reversible $\Rightarrow$ result follows METHOD 2 Show working for **(ii)** in reverse $\Rightarrow$ result follows	B1*dep B1*dep **[2]** B1* B1*dep	For correct reason. Dep on correct part(ii) For correct conclusion For correct working For correct conclusion

Question			Answer	Marks	Guidance
4	(i)				Coordinates or vectors allowed throughout
			METHOD 1 (M, then distance)		
			$M=(1+2t, 1+3t, -1+2t)$	B1	For correct parametric form soi
			$\mathbf{AM}=(\pm)[2t-6, 3t-2, 2t-8]$	B1 FT	For correct vector. FT from M
			$\mathbf{AM}$ perp. $l \Rightarrow\ 2(2t-6)+3(3t-2)+2(2t-8)=0$	M1 A1	For using perpendicular condition For correct equation
			$\Rightarrow\ t=2$, $M=(5, 7, 3)$	A1	For correct coordinates
			$AM=\sqrt{2^2+4^2+4^2}=6$	M1 A1 **[7]**	For using distance formula For correct distance
			METHOD 2(a) (distance, then M)		
			$(C=(1, 1, -1))$ $\mathbf{AC}=\pm[6, 2, 8]$	B1	For correct vector
			$\mathbf{n}=\mathbf{AC}\times[2, 3, 2]=k[-20, 4, 14]$	M1	For finding $\mathbf{AC}\times$ direction of l
			$d=\dfrac{\lvert\mathbf{n}\rvert}{\lVert[2, 3, 2]\rVert}=\dfrac{\sqrt{612}}{\sqrt{17}}=6$	A1 FT A1	For correct $\lvert\mathbf{n}\rvert$. FT from $\mathbf{n}$ For correct distance
			$CM=\sqrt{\left(6^2+2^2+8^2\right)-6^2}=2\sqrt{17}$	M1	For a correct method for finding position of M
			$\lVert[2, 3, 2]\rVert=\sqrt{17}\ \Rightarrow\ t=2$, $M=(5, 7, 3)$	B1 A1	For $\lVert[2, 3, 2]\rVert=\sqrt{17}$ soi
			METHOD 2(b)		
			$(C=(1, 1, -1))$ $\mathbf{AC}=\pm[6, 2, 8]$	B1	For correct vector
			$\cos\theta=\dfrac{AC\cdot(2,3,2)}{\lvert AC\rvert\lvert(2,3,2)\rvert}$, $\theta=36.0(39..)$ or $\sin\theta=\dfrac{153}{\sqrt{442}}$	M1,A1	
			$\lvert AM\rvert=\lvert AC\rvert\sin\theta=6$	M1,A1	
			$M=(5, 7, 3)$	M1,A1	As above

Question			Answer	Marks	Guidance
4	(ii)		$\mathbf{AM}=[-2,4,-4]$ *or* $\mathbf{MA}=[2,-4,4]$ $\Rightarrow B=(7,3,7)+\frac{3}{4}(-2,4,-4)=\left(7-\frac{3}{2},3+3,7-3\right)$ OR $B=(5,7,3)+\frac{1}{4}(2,-4,4)=\left(5+\frac{1}{2},7-1,3+1\right)$ OR $B=\frac{3}{4}(5,7,3)+\frac{1}{4}(7,3,7)=\left(\frac{15}{4}+\frac{7}{4},\frac{21}{4}+\frac{3}{4},\frac{9}{4}+\frac{7}{4}\right)$ $B=\left(\frac{11}{2},6,4\right)$	M1 M1 A1 **[3]**	For using $A+k_1\overrightarrow{AM}$ *or* $M+k_2\overrightarrow{MA}$ *or* ratio theorem *or* equivalent For $B=(7,3,7)+\frac{3}{4}$x'*their* (-2,4,-4) **oe** (**or** M1 for quadratic in parameter for line AM, followed by M1 for attempt to use **correct** value of parameter to find B) For correct coordinates
5	(i)		$\left(2m^2+3m-2=0\right)\Rightarrow m=\frac{1}{2},-2$ CF = $Ae^{\frac{1}{2}x}+Be^{-2x}$	M1 A1 **[2]**	For attempt to solve correct auxiliary equation For correct CF
5	(ii)		$\frac{dy}{dx}=pe^{-2x}-2pxe^{-2x}$ $\frac{d^2y}{dx^2}=-4pe^{-2x}+4pxe^{-2x}$ $\Rightarrow\left(-8p+3p+8px-6px-2px\right)e^{-2x}=5e^{-2x}$ $\Rightarrow p=-1$	M1 A1 M1 A1 **[4]**	For differentiating PI twice, using product rule For correct $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ For substituting into DE For correct p

Question			Answer	Marks	Guidance
5	(iii)		GS $(y=)Ae^{\frac{1}{2}x}+Be^{-2x}-xe^{-2x}$	B1 FT	For GS soi. FT from CF (2 constants) and p
			$(0,0)\Rightarrow A+B=0$	B1 FT	For correct equation. FT from GS of form $Ae^{\alpha x}+Be^{\beta x}-Cxe^{-2x}$
			$\frac{dy}{dx}=\frac{1}{2}Ae^{\frac{1}{2}x}-2Be^{-2x}-e^{-2x}+2xe^{-2x}$		
			$\left(0,\frac{dy}{dx}=4\right)\Rightarrow\frac{1}{2}A-2B=5$	M1	For differentiating GS and substituting values, using GS of form $Ae^{\alpha x}+Be^{\beta x}-Cxe^{-2x}$
			$\Rightarrow A=2,\ B=-2$	M1	For solving for A and B(can be gained from incorrect GS)
			$\Rightarrow\ y=2e^{\frac{1}{2}x}-2e^{-2x}-xe^{-2x}$	A1	For correct solution, including $\boxed{y=}$
				[5]	
6	(i)		METHOD 1		
			$\mathbf{n}=[2,-1,-1]\times[2,-3,-5]=[2,8,-4]$	M1	For finding vector product of 2 vectors in Π(or 2 scalar products = 0, with attempt to solve)
			$\mathbf{n}=k[1,4,-2]$	A1	For correct n
			Π is $\mathbf{r}.\mathbf{n}=[1,6,7].\mathbf{n}$	M1	For attempt to find equation of Π, including cartesian equation
			$\Rightarrow\ \mathbf{r}.[1,4,-2]=11$	A1	For correct equation (allow multiples)
			METHOD 2		
			$y-z=-1+2\mu$	M1	for finding λ or μ in terms of two from x,y,z.
			$\mu=\frac{y-z+1}{2}$		
			$\lambda=7-z-5\frac{y-z+1}{2}$	M1	For both λ & μ
			$x=11+2z-4y$	A1	AEF
			$r.(1,4,-2)=11$	A1	
				[4]	

Question			Answer	Marks	Guidance
6	(ii)		$[7+3t, 4, 1-t]\,.\,\mathbf{n}=11 \Rightarrow t=-2$	M1	For attempt to find t, (or to find λ and μ by equating original equations)
			$\Rightarrow [1, 4, 3]$	A1 **[2]**	For correct position vector *OR* point
6	(iii)		METHOD 1 $\mathbf{c}=[1, 4, -2]\times[2, -1, -1]$	M1	For using given vector product (or 2 correct 'scalar products = 0')
				M1	For calculating given vector product (or 2 correct scalar products = 0,with attempt to solve) (or M1 for using vector product of c with n or (2,-1,-1) in an equation, followed by M1 for calculating vector product and attempting to solve)
			$\mathbf{c}=k[2, 1, 3]$	A1 **[3]**	For correct c
			METHOD 2 $\mathbf{c}=[2, -3, -5]+s[2, -1, -1]$	M1	For c = linear combination of $[2, -3, -5]$ and $[2, -1, -1]$
			$\mathbf{c}\,.[2, -1, -1]=0 \Rightarrow$ $2(2+2s)-1(-3-s)-1(-5-s)=0$	M1	For an equation in s from $\mathbf{c}\,.[2, -1, -1]=0$
			$\Rightarrow s=-2 \Rightarrow \mathbf{c}=k[2, 1, 3]$	A1	For correct c

Question			Answer	Marks	Guidance
7	(i)		$\begin{pmatrix}1&0\\n&1\end{pmatrix}\begin{pmatrix}1&0\\m&1\end{pmatrix}=\begin{pmatrix}1&0\\n+m&1\end{pmatrix}=\begin{pmatrix}1&0\\m+n&1\end{pmatrix}$ $=\begin{pmatrix}1&0\\m&1\end{pmatrix}\begin{pmatrix}1&0\\n&1\end{pmatrix}\Rightarrow$ commutative	M1 A1 **[2]**	For multiplying 2 distinct matrices of the correct form both ways, or generalised form at least one way, For stating or implying that addition is commutative **and** correct conclusion **SR** Use of numerical matrices must be generalised for any credit
7	(ii)		$(I=)\begin{pmatrix}1&0\\0&1\end{pmatrix}$ EITHER $\begin{pmatrix}1&0\\2&1\end{pmatrix}^{-1}=\begin{pmatrix}1&0\\-2&1\end{pmatrix}=\begin{pmatrix}1&0\\4&1\end{pmatrix}$ OR $\begin{pmatrix}1&0\\2&1\end{pmatrix}\begin{pmatrix}1&0\\n&1\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}\Rightarrow 2+n=0\Rightarrow\begin{pmatrix}1&0\\4&1\end{pmatrix}$	B1 M1 A1 **[3]**	For correct identity For using inverse property For correct inverse
7	(iii)		$\begin{pmatrix}1&0\\3&1\end{pmatrix}$ has order 2 4 is not a factor of 6	B1 B1 **[2]**	For correct order For correct reason (Award B0 for "Lagrange" only). Must be explicit about the '6'
7	(iv)		$\begin{pmatrix}1&0\\1&1\end{pmatrix}$ *OR* $\begin{pmatrix}1&0\\5&1\end{pmatrix}$ has order 6, (or > 3) OR $(M,\times)$ is cyclic, G is non-cyclic (having no element of order 6) OR $(M,\times)$ is commutative G is not commutative (being the non-cyclic group) $\Rightarrow$ groups are not isomorphic	B1* B1*dep **[2]**	For stating (that there is) an element of M with order 6 Award B1* for a relevant statement about M and G For correct conclusion and no false statements attached to conclusion

Question			Answer	Marks	Guidance
8	(i)		$\cos 5\theta + \mathrm{i}\sin 5\theta =$		
			$c^5 + 5\mathrm{i}c^4s - 10c^3s^2 - 10\mathrm{i}c^2s^3 + 5cs^4 + \mathrm{i}s^5$	B1	For explicit use of de Moivre with $n = 5$
			$\Rightarrow \tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$	M1	For correct expressions for $\sin 5\theta$ and $\cos 5\theta$
			Division of numerator & denominator by c^5.		
			$\Rightarrow \tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$	M1	For $\frac{\sin 5\theta}{\cos 5\theta}$ in terms of c and s
				A1	For simplifying to **AG, www** with explicit mention of division by c^5
				[4]	
8	(ii)		$5\theta = \{1, 5, 9, 13, 17\}\frac{1}{4}\pi$	M1	For at least 2 of given values and no extras.
			$\theta = \{1, 5, 9, 13, 17\}\frac{1}{20}\pi$	A1	For at least 3 values of θ and no extras in range
				A1	For all 5 values and no extras outside range
				[3]	
8	(iii)		$\tan 5\theta = 1 \Rightarrow t^5 - 5t^4 - 10t^3 + 10t^2 + 5t - 1 = 0$	M1*	For $\tan 5\theta = 1$ and equation in t
			$\Rightarrow (t-1)(t^4 - 4t^3 - 14t^2 - 4t + 1) = 0$	A1	For correct factors
			$\tan\alpha = 1$ *OR* $\alpha = \frac{1}{4}\pi$ is not included in roots of the quartic	B1	For solution rejected (may be implied by $\frac{5}{20}\pi$ not appearing in set of solutions)
			$\Rightarrow t = \tan\alpha$ for $\alpha = \{1, 9, 13, 17\}\frac{1}{20}\pi$	M1*dep	For 2 correct values of t
				A1	For all 4 values and no more in range
				[5]	